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Quasi-Public Goods in a Tiebout Model

by

Jan K. Brueckner*

A close reading of Tiebout's famous 1956 article [2] shows that the public goods he had in mind were publicly-produced <u>private</u> goods rather than pure public goods in the Samuelson sense. For example, he states that "a doubling of the population means doubling the amount of services required." His claim that his argument addresses the problem that Samuelson identified was not therefore completely valid. It remains true, however, that publicly-produced private goods will not be optimally provided in a jurisdiction where people have different tastes for the public good and the same budget-balancing head tax is levied on all citizens. The level of the good provided is the median optimal level among voters in the jurisdiction, a level which will be non-optimal for voters with "non-median" tastes for the good.

Tiebout claimed that migration of voters to jurisdictions providing an ideal level of the public good from their point of view would occur, and that the surviving jurisdictions would provide the various public good levels at minimum unit cost. The purpose of this paper is to show formally that some of Tiebout's conclusions were correct but that broadening the

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admissable class of public goods to include other quasi-public goods, public goods subject to congestion, leads to different results on the efficiency of production in ideal jurisdictions. It is no longer true that the public good is produced at minimum average cost. The paper also shows that, except in certain special cases, the migration equilibria in the model are extremely difficult to analyse and that the possibility of the formation of homogeneous cities that are ideal from the point of view of their residents is really an open question. The paper also investigates Pareto-efficient provision of the public good.

I.

Suppose the utility functions depend on g, a numeraire private good acquired in a market and z, the consumption level of a public good provided by the government. Let the income of all consumers be y. Since the public good in Tiebout's model is actually a publicly-produced private good, $z = X/n$, where X is the production level of the good and n is the population of the jurisdiction. The good is distributed equally among all consumers in the jurisdiction. The government faces a cost function for production of the good, $C(X)$, which does not depend on the location or population of the jurisdiction and which generates a U-shaped average cost function. A jurisdiction which provides per capita consumption z to its residents must produce nz of the public good. The budget-balancing head tax for each consumer is $C(nz)/n$. The consumer's problem is to choose simultaneously levels of g, z, and n which maximize utility subject to the budget constraint. We view the consumer as choosing an ideal g, z, n combination; whether such a combination is actually available to

him is a separate issue. The Lagrangean is

$$U(g, z) - \lambda(g + \frac{C(nz)}{n} - y), \tag{1}$$

where U is some utility function and λ is a Lagrange multiplier. First order conditions are

$$U_1(g,z) = \lambda \tag{2}$$

$$U_2(g,z) = \lambda C'(nz) \tag{3}$$

$$C'(nz) = C(nz)/nz \tag{4}$$

$$g + C(nz)/n = y. \tag{5}$$

Equation (4) immediately yields $nz = \hat{X}$, where $\hat{X}$ is the average-cost-minimizing level of X output. Substituting $n = \hat{X}/z$ into (2), (3), and (5) we get the system

$$\begin{aligned} U_1(g,z) &= \lambda \\ U_2(g,z) &= \lambda C'(\hat{X}) \\ g + C(\hat{X})z/\hat{X} &= y, \end{aligned} \tag{6}$$

which solves for g, z, and λ. Knowing z^*, the consumer's optimal z, n^* can be solved for directly. The solution can be thought of as follows: consumers solve for their optimal z level and then choose the city size such that total output is provided at minimum average cost. When everyone has the same utility function, only one city size is ideal. With a variety of utility functions there will be a variety of city sizes that are ideal from the point of view of consumers. Each city has the property that $n^*z^* = \hat{X}$. Large ideal cities have low per capita consumption levels and small cities have large consumption levels. People always have an incentive to find a city where tastes match their own and production occurs efficiently. Whether such cities exist is another question entirely, as the

following problem illustrates: what if the number of people $\tilde{n}$ who find it best to live in a city with population n^* does not in fact equal n^*? If $n^* > \tilde{n}$, not enough consumers are available to populate an ideal city. If $n^* < \tilde{n}$, the number of people desiring to live in the city exceeds the ideal size. This problem will be discussed at greater length in the third section.

When X production occurs with constant returns to scale, $C(nz)/n = C(z)$ and city size drops out of the problem. People segregate themselves according to taste, but people of given taste may live in a city of any size. Constant returns to scale clearly eliminates the problem raised in the previous paragraph. When X production occurs with increasing returns to scale, equation (4) does not have a solution; people want to live in infinitely large cities.

A diagrammatic solution for the U-shaped average cost function case is shown in Figure 1. The diagram shows a family of U-shaped curves as functions of z. These curves are graphs of $C(nz)/nz$, which is the unit price of z to the consumer as well as the average cost of producing X, for different values of n. Curves corresponding to larger cities lie farther to the left, and z^* is the z level for which the demand curve cuts an average cost curve at its minimum. The optimal n is the level of n for that average cost curve. A consumer who values z more highly has a higher demand curve and wishes to consume more z in a smaller city.

II.

Suppose we have a quasi-public good, a more general class in which the publicly-produced private good is a special case. A general

characterization of such a good is the relationship

$$z = f(X,n). \tag{7}$$

A pure public good has the property that $z \equiv X$, which means $\partial z/\partial n \equiv 0$. The salient feature of a quasi-public good is that $\partial z/\partial n \equiv f_2 < 0$. More people consuming the output X leads to a reduction in per capita consumption. Also, we require $\partial z/\partial X \equiv f_1 > 0$.[1] Although no further structure on f is needed for the analysis, it is interesting to examine polar cases. In the private good case, $z = X/n$, $\partial z/\partial n = -z/n$, and $\partial z/\partial X = 1/n$, while in the pure public good case $z = X$, $\partial z/\partial n = 0$, and $\partial z/\partial X = 1$. The quasi-public good must satisfy $f_1 > 0$ and $f_2 < 0$, but a more stringent definition might require $\frac{1}{n} \leq f_1 < 1$ and $\frac{-f}{n} \leq f_2 < 0$ as well. The interpretation of these requirements is as follows: A small increase in X should increase z by an amount which is less than the X increase but not less than the X increase divided by the population. The bounds are given by the polar cases of pure public and publicly-produced private goods. Similarly, a small increase in n should have some depressing effect on z but the effect should not cause z to fall by more than $(z/n)dn$, which equals the removal of z from current citizens required to redistribute a fixed amount of a private good over a population larger by dn. These bounds on the first-order partial derivatives are speculative, but they further restrict the class of functions f which might represent the "technology" underlying quasi-public good production. One further restriction which is not needed below but seems natural is $f(X,n) < X$, which says that per capita consumption may not exceed production.[2]

By the implicit function theorem, there exists a funtion h such that

$$X = h(z,n), \tag{8}$$

with $h_1 \equiv 1/f_1 > 0$ and $h_2 \equiv -f_2/f_1 > 0$. In this framework, the consumer's problem is characterized by the Lagrangean

$$U(g, z) - \lambda(g + \frac{C(h(z, n))}{n} - y) \tag{9}$$

with first-order conditions

$$U_1 = \lambda \tag{10}$$

$$U_2 = \lambda C'(h)h_1/n \tag{11}$$

$$C'(h) = C'(h)/h_2 n \tag{12}$$

$$g + C(h)/n = y \tag{13}$$

This system leads to optimal g, z, and n, just as in the private good case. We may ask whether the solution leads to provision of the public good at minimum average cost. From (11), output occurs above $\hat{X}$, the average cost minimum, when $h_2 n < h$ at the optimum and below $\hat{X}$ when $h_2 n > h$ at the optimum. If $h_2 n = h$ at the optimum, then $X = \hat{X}$. To ascertain which case is likely to hold, we must impose further restrictions on the function h. Consider the quantity $h(\bar{z}, 0)$. This is the output level for X which provides per capita consumption $\bar{z}$ to a city of zero population. Since this is not a meaningful quantity, h is not defined for n = 0. A sensible restriction, though, is $\lim_{n \to 0} h(\bar{z}, n) = 0$ for all $\bar{z} \geq 0$, which says that as the city size approaches zero, the output level that generates a per capita consumption equal to $\bar{z}$ approaches zero as well. In terms of f, this means that f(0, 0) is not defined, and that if we consider an "f isoquant," where $f(X,n) = \bar{z}$, the limiting value of X on this curve as $n \to 0$ is also zero, which holds for all $\bar{z} \geq 0$. When this restriction is made, the question $h_2(z^*, n^*)n^* \gtreqless h(z^*, n^*)$ is reduced to a question about the sign of h_{22}. If $h_{22} > 0$, then $h_2^* n^* > h^*$, while if $h_{22} < 0$, then

$h_2{}^*n^* < h^*$, and if $h_{22} \equiv 0$, then $h_2{}^*n^* = h^*$, where $h_2{}^*$ and h^* are h_2 and h evaluated at the optimum.

Now h_{22} is $\partial^2X/\partial n^2$, which we may attempt to sign by an appeal to intuition. The essence of quasi-public goods is the congestion phenomenon: excessive numbers of people laying claim to a fixed output. To keep per capita consumption at a fixed level when claimants to the public output increase, we must increase total output: $\partial X/\partial n \equiv h_2 > 0$. The question is whether output must rise at a faster or slower rate than population to keep z constant. If for any given X and n, proportional increases in both lead to a reduction in z, then clearly $h_{22} > 0$: X must increase faster than n to keep z constant. If proportional increases in X and n lead to an increase in z, then $h_{22} < 0$: X need not be increased as rapidly as n to maintain z. "Congestion" increased (z fell) in the first case when X and n were increased proportionally, while it decreased (z rose) in the second case. Accordingly, let us refer to the $h_{22} > 0$ case as the "increasing congestion (for proportional increases in X and n)" case and label the $h_{22} < 0$ and $h_{22} = 0$ cases the "decreasing" and "constant congestion" cases respectively. Since for fixed z, the graph of h is just an f isoquant, the increasing congestion case corresponds to f being strictly quasi-concave, the decreasing congestion case corresponds to an f which is not even weakly quasi-concave, while the constant congestion case means that f is weakly, but not strictly, quasi-concave. Note that in the private good case, f is $\frac{X}{n}$, which is weakly, but not strictly, quasi-concave. Equivalently, since h is nz in this case, $h_{22} = 0$. The private good is a particular example of the constant congestion case.

Now h_{22} equals

$$-f_1^{-3}(f_{11}f_2^{2} - 2f_{12}f_1f_2 + f_{22}f_1^{2}), \tag{14}$$

which is just the "marginal rate of transformation" along an f isoquant. The assumption $f_{11} < 0$, diminishing returns to X for fixed n, seems reasonable. The further assumptions $f_{12} < 0$ and $f_{22} < 0$, which are also apparently reasonable requirements, yield $h_{22} > 0$. But $f_{22} < 0$, which means that for fixed X, z decreases at a increasing rate as n increases, does not hold in the private good case, where $f_{22} = 2Xn^{-3} > 0$. This suggests that there are other sensible f functions which violate the above requirements, so we refrain from imposing them. Although we cannot sign h_{22}, we have proved the following:

Theorem 1: If the publicly-produced good is a quasi-public good in that $z = f(X,n)$, where $f_1 > 0$ and $f_2 < 0$, if equal city-budget-balancing head taxes are levied on each consumer, and if the total cost function for X generates a U-shaped average cost curve, then the following is true when $\lim_{n \to 0} h(\bar{z}, n) = 0$ for all $\bar{z} \geq 0$:

> Regardless of the nature of his utility function, the consumer's ideal city has an X output such that $X \lesseqgtr \hat{X}$ as $h_{22} \gtreqless 0$; X is less than, equal to, or greater than the AC minimizing output when the increasing, constant, or decreasing congestion cases, respectively, obtain.

See Figure 2 for a diagrammatic illustration of the theorem.

The following problem may arise, as happened above: a certain number of consumers $\hat{n}$ may have utility functions which dictate that their ideal community size is $n^* \neq \hat{n}$. This problem is discussed further in the next section. We saw that constant returns to scale eliminated this difficulty in the private good case. However, from (12) it is evident that no solution to the consumer problem exists with constant returns to scale and increasing or decreasing congestion: $h \neq h_2 n$ for all z and n means AC $\neq$ MC at the optimum, which is impossible with constant returns to scale, where MC $\equiv$ AC.

In the constant returns to scale with constant congestion case, city size drops out of the problem. Since $h_2(z, n)n \equiv h(z, n)$, integration yields $h(z, n) = A(z)n$, and $C(h)/n$ becomes $C(A(z))$. The private good case was a particular example of this. A solution with increasing returns to scale appears to be possible in the increasing congestion case, where $h_2 n > h$. Then (12) says MC < AC, which is consistent with increasing returns.

An interesting final point concerns pure public goods. The head tax in a community providing a level X of the pure public good is $C(X)/n$. Solving for the optimal n in this framework is impossible since the head tax decreases monotonically as n increases without bound. People desire to live in arbitrarily large communities in order to spread the cost of the public good, which is not subject to congestion, over as large a population as possible.

III.

It has been established that ideal communities need not produce at minimum average cost when the publicly-produced good is a general quasi-public good. Now we may ask whether ideal communities are indeed formed in a world with zero migration costs. In the constant returns to scale, constant congestion case, ideal homogeneous communities will be formed in any array of sizes. But in general, formation of ideal communities is less certain. If there are m types of individuals, with $k_i n_i$ consumers of the ith type, where k_i is a non-negative integer, and if by accident the optimal city size for the ith type, n_i*, happens to equal n_i, $i = 1, \ldots, m$, then k_i ideal homogeneous communities of type i individuals will be formed, $i = 1, 2, \ldots, m$. However, there is no reason to expect such a fortuitous situation. Suppose there are two types of individuals, with

ideal city sizes n_1* and n_2*, and populations n_1 and n_2 such that $2n_1^* > n_1 > n_1^*$, and $n_2 < n_2^*$. Can an ideal city be formed in this situation? Suppose that $n_2 > n_1 - n_1^*$, and suppose we require one ideal type 1 city (A) and another mixed city (B), whose median voter will be a type 2 consumer. For this situation to be locally stable it must be true that a type 1 consumer moving to A reduces his utility after a new z level, based on the now larger population, has been chosen in A. The same must be true for a type 2 moving from B. But satisfaction of these requirements does not imply that other locally stable equilibria do not exist. It appears that ideal homogeneous cities would emerge only by accident. The extreme complexity of situations where the number of consumers of different types is not "right" should be obvious.

We might imagine, though, that if the number of type i consumers were extremely large compared to n_i* for all i, an equilibrium might approximate the one with the "perfect fit" that was described above. Numerous near-ideal cities might be observed; we might imagine extra type i consumers distributed over a large number of type i cities, making each one slightly too big. Whether or not these cities would produce the good at minimum average cost is uncertain because of their non-ideal size.

IV.

In the preceeding analysis, supply considerations have not played a role in that no restrictions were placed on aggregate consumption in the city. For instance, the city did not need to trade off X if its residents wished to consume more g. Suppose, however, that cities are closed to trade with only local resources such as labor available for g and X production. Then a city of size n will have a transformation curve defined by F(X, G, n) = 0, where G is total output of g and the n argument

captures the influence of labor force size on the position of the transformation curve.

Suppose the government wishes to maximize a social welfare function linear in the utilities of the city residents, who need not be identical. The Lagrangean for such a problem is

$$\Sigma\lambda_i U^i(g_i, z) - \mu(z - f(x, n)) - \tau(\Sigma g_i - G) - \gamma F(X, G, n), \tag{15}$$

where λ_i are the welfare weights and μ, τ, and γ are Lagrange multipliers. The first order conditions yield

$$\Sigma\, U_2{}^i/U_1{}^i = F_1/f_1F_2 \tag{16}$$

which says that the sum of the marginal rates of substitution between g and z must equal the marginal rate of transformation between G and z. This, of course, is also the condition for Pareto efficiency. For illustrative purposes, we may set $\lambda_i = 1$ for all i and assume that utility functions are all identical. Then (16) is

$$U_2/U_1 = F_1/f_1F_2n. \tag{17}$$

If the government sets the price of z at $t^*(n)$, which equals F_1/f_1F_2n evaluated at the optimum, and if it adjusts per capita income so that each consumer's income equals $y^*(n) = g^*(n) + t^*(n)z^*(n)$, where $g^*(n)$ and $z^*(n)$ are the socially optimal g and z, individual utility maximization will generate the socially optimal outcome when both g and z are normal goods. This is true because normality implies that the implicit relationship $g = B(z)$ which results from $U_2/U_1 = t^*(n)$ has the property that $B' > 0$. This fact, in conjunction with the downward-sloping budget constraint, $g + t^*(n)z = y^*(n)$, guarantees a unique solution at $z^*(n)$, $g^*(n)$. Each consumer votes for the public good level $z^*(n)$, which is then provided by the government.

If all consumers are identical and all cities are run efficiently, the consumer's choice-of-city problem reduces to a choice of city size: the consumer maximizes $U(g^*(n), z^*(n))$ with respect to n. If the number of consumers is large compared to the ideal city population, an equilibrium might result with many efficiently-run cities, each with approximately the ideal population.

The tax revenue in a city of size n is

$$nz^*(n)t^*(n) = fF_1/f_1F_2 \tag{18}$$

where the right hand side is evaluated at the optimum. We may ask whether tax revenues pay for provision of the public good.[3] As long as production of X and g is cost minimizing, F_1/F_2 will equal the ratio of the marginal costs of production for X and g, MC_X/MC_G. Perfect competition among g producers means marginal cost equals average cost which equals the price of g, which is unity. So the absolute value of the slope of the transformation curve equals MC_X. Suppose X is produced by the government at constant returns to scale or that it is purchased by the government from perfectly competitive producers. Then $MC_X = AC_X$ and from (18) tax revenue is $(f/Xf_1) \cdot (XAC_X)$, which exceeds the total cost of the public good when $f > Xf_1$ and falls short of total cost when $f < Xf_1$. Since $f(0, n) = 0$ is a natural requirement, these inequalities are equivalent to $f_{11} < 0$, $f_{11} > 0$. We have

Theorem 2: In an efficiently run city with identical consumers where the government maximizes a linear social welfare function with equal welfare weights, production of g is perfectly competitive, and X is produced either by perfectly competitive producers or by the government with constant returns to scale, then tax revenues are greater than (less than) the cost of providing the public good when there are decreasing (increasing) returns to scale for X in generation of z, per capita consumption of the public good.

If the government produces the good with a non-constant returns to scale production function, MC_X is not necessarily equal to AC_X, and the Theorem does not hold.

In the analysis of sections I and II, the government was viewed as producing the public good itself. If the government purchases X from a perfectly competitive industry, then $C(X)/X$, the average cost curve, must be interpreted as a long run industry supply curve.

V.

This paper has shown that Tiebout's conjecture that ideal cities produce the good at minimum average cost was correct when the publicly-produced good is a private good, but it has been demonstrated that enlarging the class of admissible goods to include all congestible public goods yields ideal cities that do not necessarily produce at the average cost minimum. In addition, the explicit formulation of the consumer choice problem has suggested that migration equilibria may not result in the formation of ideal cities, as Tiebout believed. In analyzing Pareto-efficient provision of the quasi-public good, it has been shown that total tax revenue generally will not equal the cost of providing the public good when consumers are identical. The characterization of quasi-public goods employed in the analysis has generated a number of interesting results; fruitful use of this concept in other areas of public finance also may be possible.

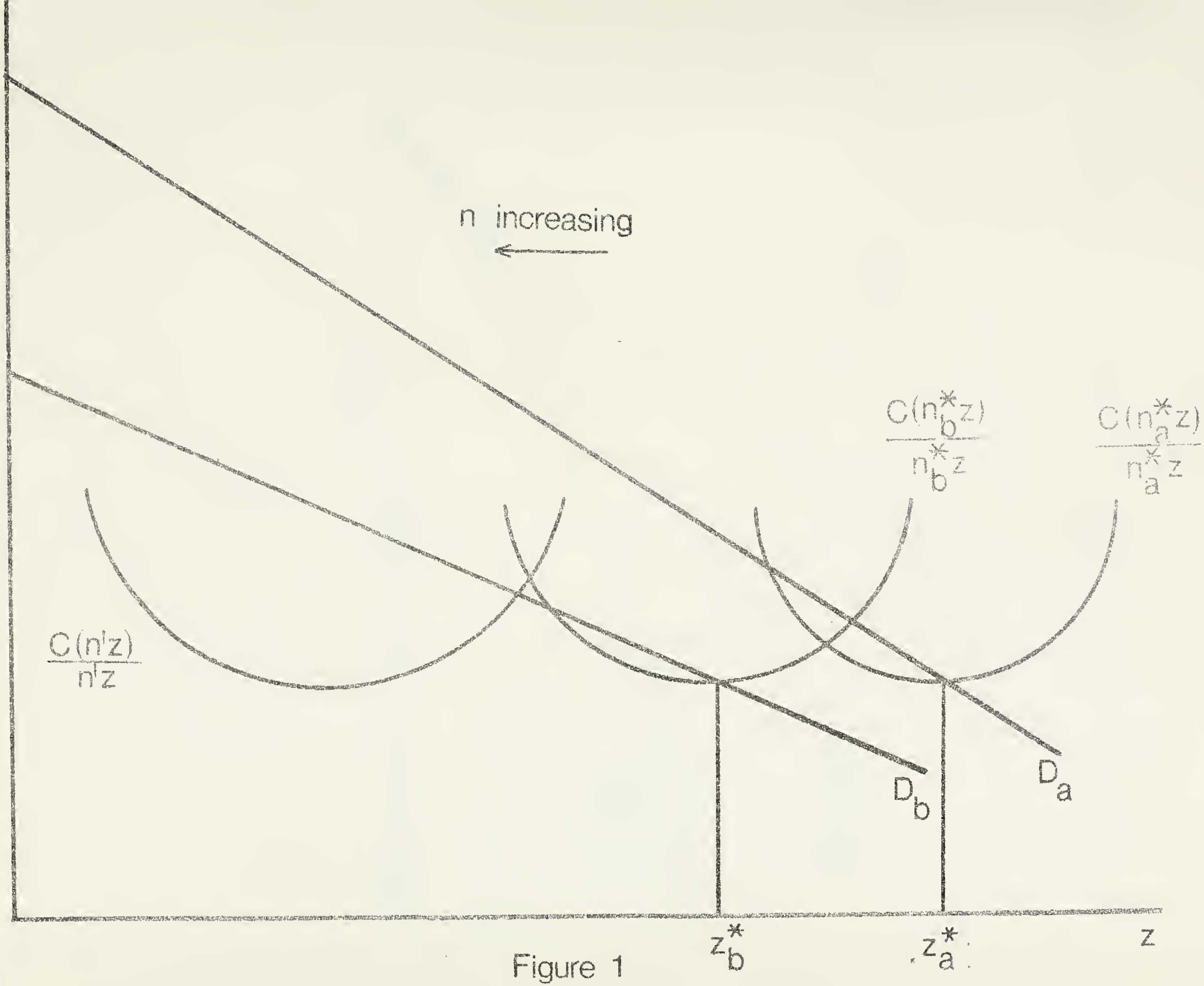

Figure 1

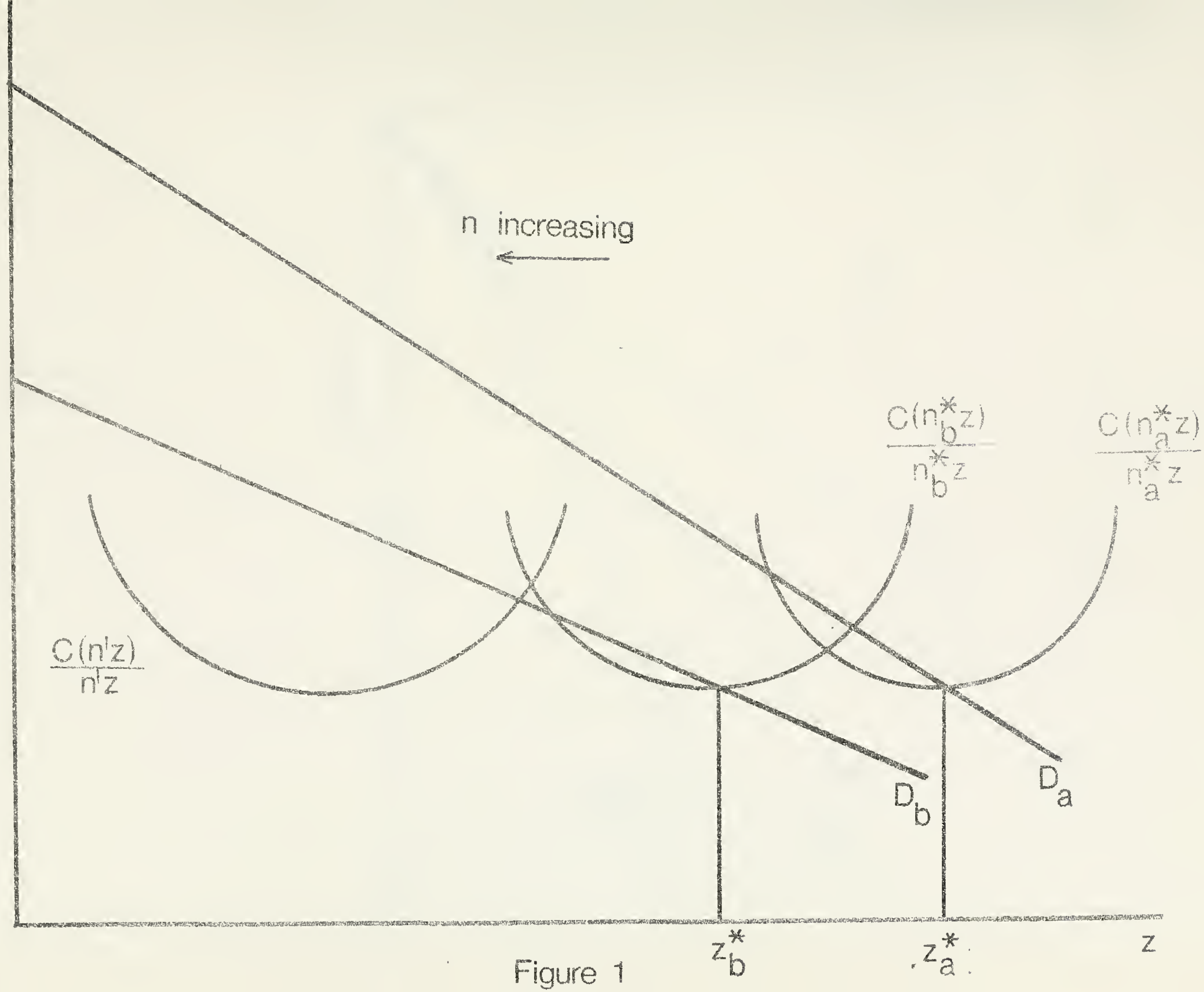

Figure 1

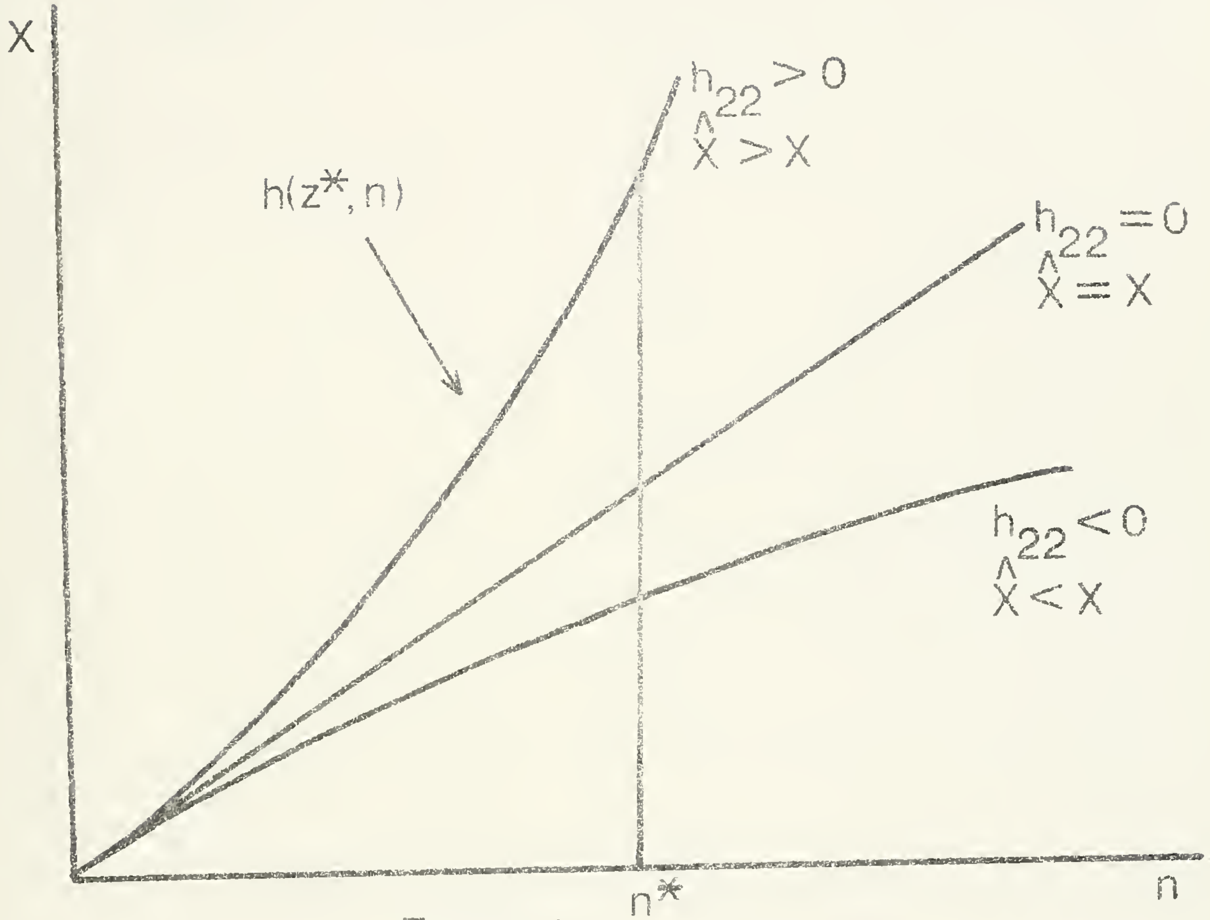

Figure 2

REFERENCES

1. Oakland, W. H., "Congestion, Public Goods, and Welfare," Journal of Public Economics, November, 1972.

2. Tiebout, C. "A Pure Theory of Local Expenditures, "Journal of Political Economy, October, 1956.

FOOTNOTES

[1] While an example of a publicly-produced private good might be water supplied to a municipality by the government, an example of a quasi-public good that is not private would be police or fire services. Output could be police or fire units on duty and per capita "consumption" could be average response time. For fixed output, response time falls as n increases. Some other common public goods are less well characterized by (8). We might imagine that per capita "consumption" of parks would depend not only on acres of parks provided and population but also on the tastes of the population for parks. Parks in a city of park lovers would be more congested than in another city with the same park acreage and population but fewer park enthusiasts. For a park-like public good, it seems that the tastes as well as the size of the population must enter a relationship such as (8).

[2] See Oakland [1] for a different treatment of congestible public goods.

[3] Some steps in the following analysis draw on Oakland [1].